Experimental Analysis of a Passively Tuned Actuator on a Low-Order Structure

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This paper presents results of a study of the interaction between a passively tuned reaction mass actuator and a low-order structure. The actuator was passively tuned to the structure's first resonant frequency, and the uncontrolled and passively damped structural responses were compared. While introducing a new mode in the system, the actuator significantly reduced the peak amplitude ratios of the first two structural modes. The peak amplitude ratios of the structure's higher modes were reduced as well. A sensitivity analysis of the actuator's tuning showed that the passively damped system's resonant modes were altered if the actuator was not tuned to the optimum frequency and damping ratio. Finally, the actuator reduced the magnitude of the structural vibrations, not only where the actuator was attached to the structure, but at all points on the structure.

Introduction

In the near future, large flexible space structures will become a reality. One requirement common to most of these structures will be the need to minimize unwanted vibrations. Techniques to reduce the vibrations of a flexible structure can be separated into two categories: passive damping and active control. Although the best way to control a structure's vibrations is to use a combination of passive damping and active control techniques,[1] this paper explores only the effects of passive damping on a structure.

The need to passively damp a large space structure's vibrations has been well established,[2,3] and various ways to introduce passive damping have been thoroughly examined.[4-7] One technique uses an actuator tuned to one of the structure's natural frequencies. The effectiveness of this damping method may be determined by analyzing the extent of vibration reduction on the structure. Further, the sensitivity of the structure to variations in the tuning of the actuator must be examined.

This paper uses analytical and experimental techniques to examine the interactions between a passively tuned reaction mass actuator and a flexible structure to which it is attached. A finite element model of a flexible structure possessing coupled modes at low frequencies was generated and then verified with an experimental structure. Using these models, a study was conducted to examine the interactions between the structure and the actuator. Specifically, the extent to which the actuator reduced the structure's vibrations was analyzed, and the sensitivity of the structure to a mistuned actuator was examined.

Analytical Model

The flexible structure used in this study was designed to possess low-order coupling—there were to be two dominant low-frequency structural modes on the order of 5 Hz. In addition, the higher modes were to be at frequencies at least three times that of the first mode to limit the number of low-frequency resonances to two.

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A 48-degree-of-freedom finite element model of this structure was generated using MSC/NASTRAN. The first three resonant frequencies of this analytical model were 5.198, 6.160, and 16.622 Hz. The structure was designed such that these frequencies could be altered, if so desired. The sketch of the system in Fig. 1 shows that there are two brackets on the horizontal beam—these were supports for two reaction mass actuators. Only one actuator was used in this study. By positioning the brackets at outboard positions along the horizontal beam, the first two modes could be decoupled from each other. Conversely, the modes could be closely coupled together by simply moving the brackets to particular symmetric inboard positions. For the configuration illustrated in Fig. 1, the structure's first resonant frequency corresponded to a torsional or twisting motion about the z axis. The second frequency corresponded to a bending motion about the y axis. These shapes indicated that the only nodal point along the horizontal bar was at the center. The stiffening effect of gravity on the vertical members was included in the model, which had about a 3% effect on the model's natural frequencies.

Because a higher-order analytical model would have been difficult to use due to memory limitations on the computer that contained the MATLAB program, the NASTRAN model was reduced to a fourth-order system using an accurate reduction technique outlined by Hallauer and Barthelemy.[8] Undesired columns and rows were eliminated from the mass normalized modal matrix of the full model: four degrees of freedom and four modes were identified to be retained, and all other matrix elements were eliminated. The reduced modal matrix was mass normalized since the original modal matrix was also mass normalized. Then,

$$P_r^t M P_r = I \tag{1}$$

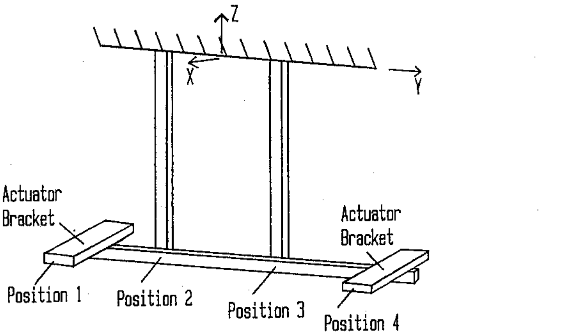

Fig. 1 Model of low-order structure.

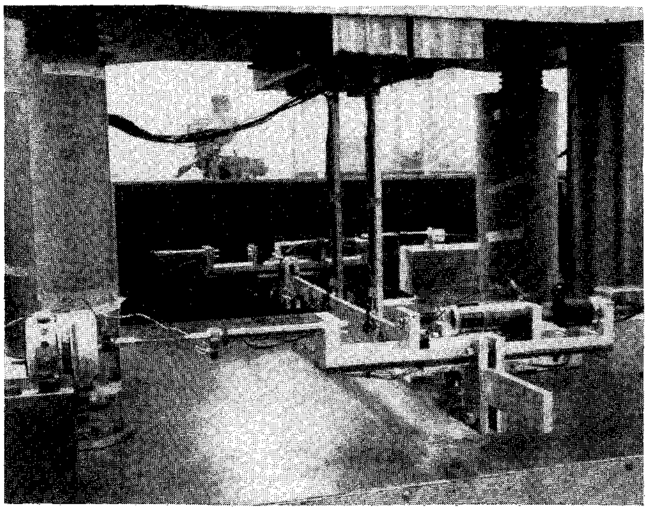

Fig. 2 Experimental low-order structure.

$$M = [P_r']^{-1}P_r^{-1} \tag{2}$$

$$K = [P_r']^{-1}[\omega^2][P_r]^{-1} \tag{3}$$

where P_r was the reduced modal matrix and $[\omega^2]$ was the reduced diagonal eigenvalue matrix. The reduced equation of motion was then

$$[M]\{z''\} + [K]\{z\} = \{0\} \tag{4}$$

where the elements of the vector z were the displacements of the system's four degrees of freedom. The reduced M and K matrices were accurate as long as P_r was not ill conditioned. The eigenvalues and eigenvectors of the reduced model were checked for agreement with the full model. This reduced model could then be used to determine the actuator's optimal passive tuning parameters with an existing MATLAB routine.

Experimental Structure

The analytical model was then validated by assembling an experimental structure. This structure, shown in Fig. 2, was constructed of aluminum beams; the vertical members were $0.375 \times 1.5 \times 15.625$ in. and the horizontal member was $0.375 \times 1.5 \times 50$ in. The brackets for the reaction mass actuators (RMAs) used in this study were constructed of aluminum and structural steel and were located 18 in. from the structure's centerline. The baseline, or open-loop, structure was defined to consist of no moving parts. Hence, each RMA, in the baseline configuration, consisted of the following parts: a bracket structure, a linear velocity transducer (minus a moving shaft), a Kaman displacement sensor, and a coil for the RMA. The inertial properties of each baseline actuator bracket assembly were the following: mass = 6.30 lb, $I_{xx} = 9.40$ lb-in.^{2}, $I_{yy} = 92.40$ lb-in.^{2}, and $I_{zz} = 94.30$ lb-in.^{2}. The center of gravity of each bracket was 0.9 in. above the centerline of the horizontal member.

To determine the structure's resonant frequencies, a Tektronix 2630 Fourier Analyzer generated a random signal that was amplified and sent through a Ling Model V102 shaker to vibrate the structure at position 4 (see Fig. 1). The force output from the shaker was measured with a PCB force gauge, and the structure's relative position was measured with several electromike displacement sensors. The Fourier Analyzer generated frequency response functions of the structure for responses at position 4. Table 1 compares the structure's analytical and experimental resonant frequencies. Note the close correlation between the two sets of data for the first four modes. With both models in agreement, passive tuning of the RMA could be accomplished.

Table 1 Analytical and experimental structural resonant frequencies

Analytical, Hz	Experimental, Hz	Percent error, %
5.198	5.162	0.690
6.160	6.063	1.581
16.622	16.069	3.328
24.373	24.950	2.313

Analytical Tuning of the Actuator

The method used to determine the frequency and damping ratio to optimally tune the actuator was adapted from Ref. 9, which described a method for calculating the optimal tuning of an actuator attached to a multi-degree-of-freedom system. In this method, the structure was disturbed where the actuator was attached to the structure. The actuator and shaker were attached to position 4 for this study.

With the fourth-order mass and stiffness matrices from the reduced NASTRAN model, the RMA was optimally tuned to the first resonant frequency of the analytical model using the following equations[9]:

$$\bar{\omega}_{\text{opt}} = k_\omega\left[\frac{1}{1+\mu}\right] \tag{5}$$

$$\bar{\zeta}_{\text{opt}} = k_\zeta\left[\frac{3\mu}{8(1+\mu)^3}\right]^{1/2} \tag{6}$$

where μ is the ratio of the actuator mass to the modal mass of the first structural frequency, $\bar{\omega}_{\text{opt}} = \omega_{\text{opt}}/\omega_1$ is the optimum frequency ratio to tune the actuator (ω_{opt} is the optimum natural frequency and ω_1 is the frequency of the first structural mode), ζ_{opt} is the optimum damping ratio for the actuator, and k_ω, k_ζ are correction factors to obtain the optimum tuning parameters.

The mass ratio for the analytical model was $\mu = 0.25335$, which, admittedly, is not a realistic mass ratio for proposed space structures. However, since the mass of the RMA was 4 lb, the structure would have weighed about 400 lb to obtain a desired mass ratio of 0.01. Since this was too large a structure for current laboratory conditions, the design was considered to be adequate for this investigation.

With k_ω and k_ζ numerically calculated to be 0.904 and 1.080, respectively, the optimum tuning parameters were $\bar{\omega}_{\text{opt}} = 0.72127$ ($\omega_{\text{opt}} = 3.749$ Hz) and $\zeta_{\text{opt}} = 0.23724$. Figure 3 is a comparison between the frequency response functions of the baseline and optimally tuned structures.

Notice that the actuator, when passively tuned to the structure's first resonant frequency, introduced another resonant mode to the system due to the additional degree of freedom. In addition, the actuator altered the structure's first mode: the frequency was shifted higher and the peak amplitude ratio was reduced, and there was approximately a 94.7% reduction when compared to the baseline mode. The actuator also altered the structure's second resonant mode by raising the frequency and reducing the peak amplitude ratio; again, there was about a 95% reduction in the peak amplitude ratio. The third and fourth modes were not significantly altered with the addition of the RMA.

These results can be physically interpreted as follows. Since the structure's amplitude at resonance is limited by damping, keeping the amplitude small requires damping energy to be dissipated. An optimally tuned, passively damped actuator uses the relative motion between the structure and actuator to dissipate the energy. Optimal tuning produces a lightly damped actuator that guarantees relative motion and, hence, dissipation of energy. The farther the structure's resonances are from the resonant frequency of the actuator, the less energy that is dissipated.[10] As was noted previously, the only nodal point along the model's horizontal bar was at the center. Although placing the actuator there would have helped to damp the structure's bending resonance, it would have had no effect on damping the torsional resonance. Since the actuator was located at the end of the horizontal bar, both structural resonances were damped.

To investigate the sensitivity of the structure to a mistuned actuator, the RMA was tuned to 80 and 120% of the calculated optimum frequency. Figure 4 shows the optimally tuned structural response, as well as the structural responses for the mistuned cases. The actuator's damping ratio was maintained at the optimum value in each case.

Clearly, mistuning the actuator alters the structure's resonance characteristics. At 80% of the optimum frequency, the resonant frequencies were lower than the optimum case. The peak amplitude ratio of the actuator was less than the optimum, whereas the amplitudes of the two structural modes were higher. With the actuator tuned to 120% of the optimum frequency, the resulting resonant frequencies were higher than optimum. Further, the peak amplitude ratio of the actuator's mode was higher than optimum, and the peak amplitude ratios of the two structural modes were lower. These results illustrate the sensitivity of the system to variations in the actuator's tuning.

Experimental Tuning of the Actuator

The analytical results were verified experimentally by attaching an RMA to the structure. The RMA assembly is shown in detail in Fig. 5. The moving mass core was built around a series of rare Earth magnets that encircled the RMA coil. A magnetic shaft, connected to this core, passed through the center of the stationary coil of the linear velocity transducer and sensed the velocity of the reaction mass relative to the structure. The noncontacting Kaman transducer sensed the relative position of the reaction mass. The resulting analog signals from the transducers were fed back to the RMA to effect electromagnetic stiffness and viscous damping on the RMA. The RMA's natural frequency and damping ratio could be varied by adjusting two feedback gain potentiometers to their desired values. There was a small amount of uncontrollable rolling friction in the motion of the RMA shaft through two linear bearings; hence, the total damping of the RMA was a combination of this small, but uncontrollable, friction plus the controllable and much larger linear viscous damping.

The RMA was tuned to the optimum parameters calculated analytically: $\bar{\omega}_{opt} = 0.72127$ ($\omega_{opt} = 3.723$ Hz since the experimental structure's first resonant frequency was 5.162 Hz) and $\zeta_{opt} = 0.23724$. A comparison of the uncontrolled and passively tuned structural frequency responses is shown in Fig. 6.

As with the analytical results, the passively tuned RMA introduced another resonant mode to the system's frequency spectrum. The first two structural modes were shifted higher than the baseline response, and the peak amplitude ratios of these modes were reduced; specifically, the first mode's peak amplitude ratio was reduced by 58.3% and the second was reduced by 66.7%. It should be noted that the percent reduction in the peak amplitude ratios is not as great as those obtained analytically. This discrepancy is a result of not in-

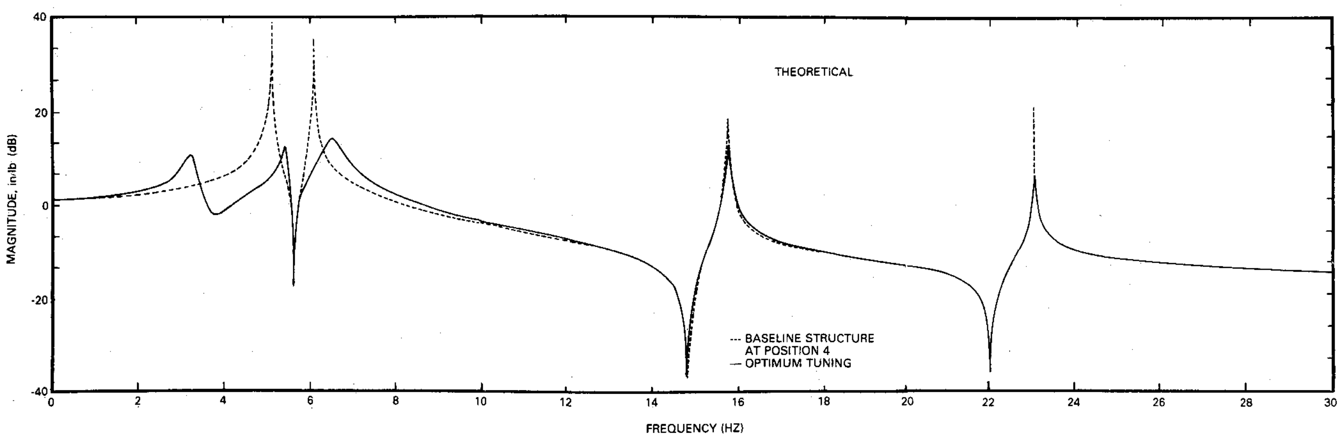

Fig. 3 Analytical baseline and optimally tuned structural response.

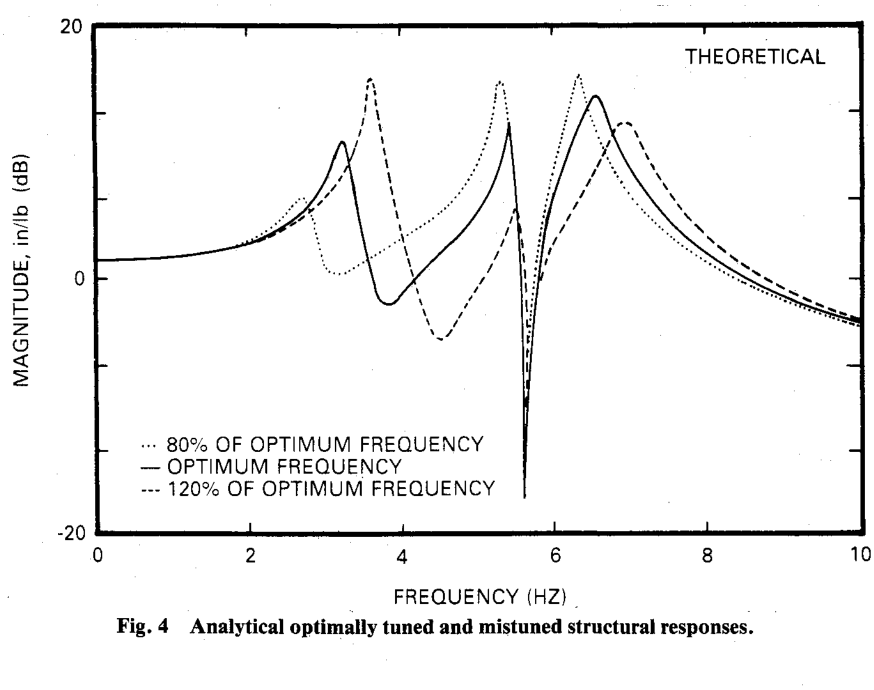

Fig. 4 Analytical optimally tuned and mistuned structural responses.

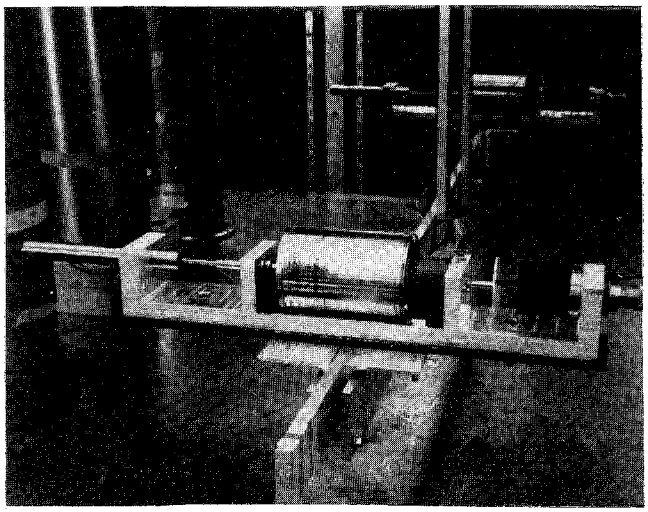

Fig. 5 Reaction mass actuator assembly.

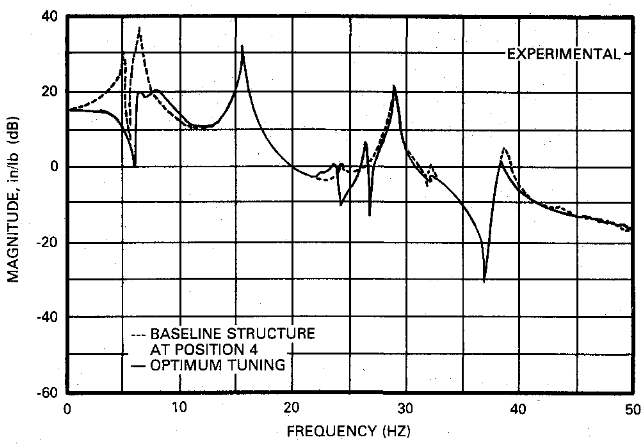

Fig. 6 Experimental baseline and optimally tuned structural responses at position 4.

cluding damping in the analytical model of the structure. Structural damping would reduce the peak amplitude ratios of the analytical frequency responses; the reduction in the baseline response would be larger, thus decreasing the 95% reduction in peak amplitude ratios reported previously.

Figure 6 also shows that the structure's higher modes were not significantly altered by adding, to the structure, an actuator that was passively tuned to the structure's first resonant frequency. This mirrors the results obtained analytically.

The sensitivity of the structure to a mistuned actuator can be observed experimentally by tuning the RMA to 80 and 120% of the optimum frequency, as shown in Fig. 7. As with the analytical study, the damping ratio was kept at the optimum value in all cases. The experimental results once again agreed with those obtained analytically. At 80% of the optimum frequency, the first three resonant frequencies of the system were lower than the optimum response. The peak amplitude ratio of the actuator's mode was less than the optimum case, whereas the peak amplitude ratios of the first two structural modes were larger. With the actuator tuned to 120% of the optimum frequency, the first three resonant frequencies were higher than optimum. The peak amplitude ratio of the actuator's mode was greater than in the optimum case and the first two structural modes had peak amplitude ratios less than the optimum case. In either situation, higher modes were not altered from the optimum frequency spectrum.

Based on the results obtained analytically and experimentally, a reaction mass actuator passively tuned to the structure's first resonant frequency generated a new mode in the

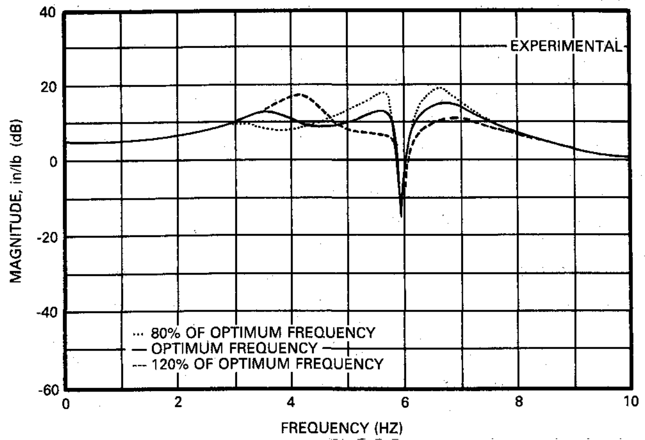

Fig. 7 Experimental optimally tuned and mistuned structural responses at position 4.

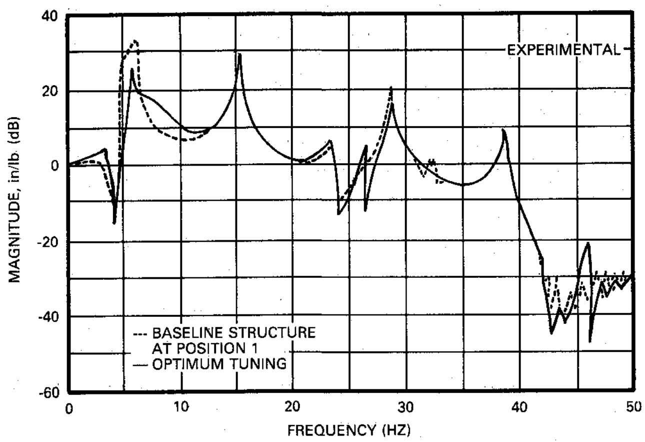

Fig. 8 Experimental baseline and optimally tuned structural responses at position 1.

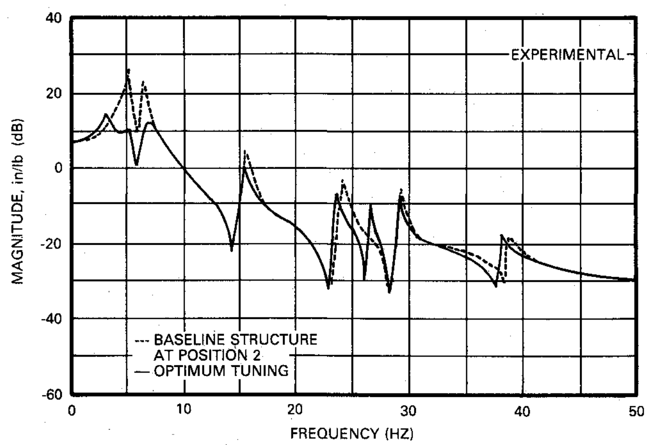

Fig. 9 Experimental baseline and optimally tuned structural responses at position 2.

system but at the same time, significantly reduced the peak amplitude ratios of the structure's first two resonant frequencies. Passive damping did not alter higher modes; in fact, the farther away the second resonant mode was from the first, the less of an effect the actuator had on that mode. A mistuned actuator did alter the structure's resonant modes, particularly those close to the actuator's mode.

Effects on Other Points on the Structure

Because the shaker assembly, electromike position sensor, and RMA were all located at position 4, these results are valid

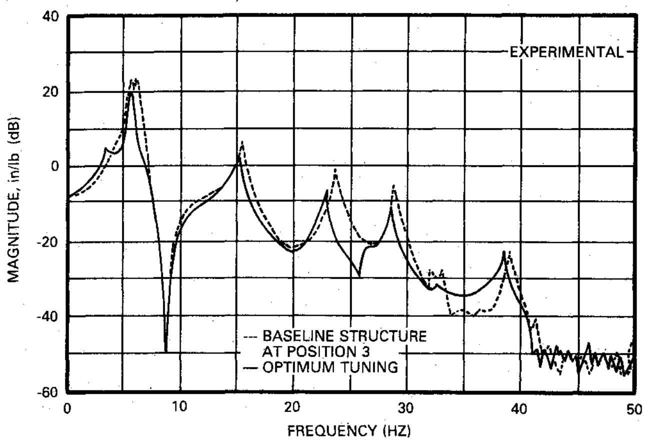

Fig. 10 Experimental baseline and optimally tuned structural responses at position 3.

only for situations where the control actuator, disturbance force, and displacement sensor of the structure are collocated. To see if the actuator damped the structural vibrations on other points on the structure, the electromike position sensor was placed at various locations on the structure and structural frequency response functions were obtained.

The structural responses at positions 1, 2, and 3 (see Fig. 1) are shown in Figs. 8–10 for both the baseline configuration and the structure with the RMA passively tuned to the structure's first resonant frequency and attached at position 4. The figures show that the actuator acted as a passive vibration absorber for the entire structure, even though it was attached to a single point on the structure. As previously observed, the RMA had a greater effect on the lower modes; the actuator's ability to reduce the structure's vibrations diminished farther from the resonant frequency of the RMA.

Conclusions

Based on the analytical and experimental results presented, it is evident that a reaction mass actuator, passively tuned to the optimum frequency and damping ratio of a structure's first resonant frequency, reduces the peak amplitude ratios of the structure's first few resonant modes. The effects diminish for higher modes farther from the actuator's tuned frequency. The damping of the structure is sensitive to the tuning of the actuator since the structure's resonant frequencies and peak amplitude ratios are altered if the actuator is mistuned from the optimum frequency. Anywhere on the structure, the optimally tuned reaction mass actuator reduces the peak amplitude ratios of a structure's resonant modes that are close to the actuator's tuned frequency.

Acknowledgments

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References

[1]Lynch, P. J., and Banda, S. S., "Active Control for Vibration Damping," *Large Space Structures: Dynamics and Control,* edited by S. N. Alturi and A. K. Amos, Springer-Verlag, Berlin, 1988, p. 239.

[2]Ashley, H., and Edberg, D. L., "On the Virtues and Prospects for Passive Damping in Large Space Structures," *Damping 1986 Proceedings,* Air Force Wright Aeronautical Labs., AFWAL-TR-86-3059, Wright-Patterson AFB, OH, 1986, pp. DA-1–DA-17.

[3]Skelton, R. E., "Algorithm Development for the Control Design of Flexible Structures," *Proceedings from the Modelling, Analysis, and Optimization Issues for Large Space Structures,* NASA CP-2258, 1982.

[4]Chen, G.-S., and Wada, B. K., "Passive Damping for Space Truss Structures," *Proceedings of the 29th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference,* AIAA Paper 88-2469, Washington, DC, May 1988.

[5]Juang, J.-N., "Optimal Design of a Passive Vibration Absorber for a Truss Beam," *Journal of Guidance, Control, and Dynamics,* Vol. 7, No. 6, 1984, pp. 733–739.

[6]Miller, D. W., and Crawley, E. F., "Theoretical and Experimental Investigation of Space-Realizable Inertial Actuator for Passive and Active Structural Control," *Journal of Guidance, Control, and Dynamics,* Vol. 11, No. 5, 1988, pp. 449–458.

[7]Hagood, N. W., and Crawley, E. F., "Experimental Investigation into Passive Damping Enhancement for Space Structures," *Proceedings of the 1989 AIAA Guidance, Navigation, and Control Conference,* AIAA Paper 89-3436, Washington, DC, 1989.

[8]Hallauer, W. L., Jr., and Barthelemy, J.-F. M., "Active Damping of Modal Vibrations by Force Apportioning," *Proceedings of the 21st AIAA Structures, Structural Dynamics, and Materials Conference,* AIAA Paper 80-0806, New York, 1980, pp. 863–873.

[9]Duke, J. P., Webb, S. G., and Vu, H., "Optimal Passive Control of Multi-Degree of Freedom Systems Using a Vibration Absorber," *Proceedings of the 1990 AIAA Guidance, Navigation, and Control Conference,* AIAA Paper 90-3499, Washington, DC, 1990.

[10]Den Hartog, J. P., *Mechanical Vibrations,* 4th ed., McGraw-Hill, New York, 1956, p. 97.